Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2015-2016
End-Semester Examination : Statistics (I)

Date: 09-11-2015 **Maximum Score 50** **Duration: 3 Hours**

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from the following *probability density function (pdf)*
$$f_{\mathrm{X}}(x|a,b) = abx^{a-1}(1-x^a)^{b-1} I_{(0,1)}(x), \tag{1}$$
where $a > 0$, $b > 0$. Obtain the k-th moment of the distribution. If $a > 0$, $b > 0$ in (1) are both known, then how would you draw a random sample of size 11 from (1) using a **direct** method?

[6 + 9 = 15]

2. Let $X_1, X_2, \cdots, X_n$ be a random sample from the distribution with *pdf* given by
$$f(x|\theta_1, \theta_2) = \frac{1}{\theta_2 - \theta_1} I_{(\theta_1, \theta_2)}(x);\ \theta_1 < \theta_2 \in \mathbb{R}$$
Obtain *maximum likelihood estimators (MLE)* for θ_1 and θ_2.

[10]

3. Let X_1, X_2, X_3 be *independent Poisson*(λ_i) random variables, $\lambda_i > 0$, $i = 1, 2, 3$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + X_3$. Let further $V = I_{\{0\}}(Y_1)$ and $W = I_{\{0\}}(Y_2)$. Find ρ_{VW}, the correlation coefficient between V and W.

[10]

4. Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where σ^2 is unknown. Derive the *likelihood ratio test* (*LRT*) for testing the hypotheses
$$H_0 : \mu = \mu_0 \ versus\ H_1 : \mu \neq \mu_0.$$
Also obtain the *p-value.*

[10]

5. Let X be the number of defects in printed circuit boards. A random sample of $n = 60$ printed circuit boards is taken and the number of defects recorded. The results were as follows:

No of defects	0	1	2	3
Observed Frequency	32	15	9	4

Does the assumption of a Poisson distribution seem appropriate as a model for these data? Take $\alpha = 0.05$. Also obtain the *p-value.*

[15]